Just as any two points in the Cartesian plane with different x values and different y values determine a unique slant line and its related linear function, any three noncollinear points with different x values determine a quadratic function. In general, $(n + 1)$ points positioned with sufficient generality determine a polynomial function of degree n. The process of fitting a polynomial of degree n to $(n + 1)$ points is **polynomial interpolation**. Exploration 2 involves two polynomial interpolation problems.

EXPLORATION 2 Interpolating Points with a Polynomial

1. Use cubic regression to fit a curve through the four points given in the table.

x	-2	1	3	8
y	2	0.5	-0.2	1.25

2. Use quartic regression to fit a curve through the five points given in the table.

x	3	4	5	6	8
y	-2	-4	-1	8	3

How good is the fit in each case? Why?

Generally we want a reason beyond "it fits well" to choose a model for genuine data. However, when no theoretical basis exists for picking a model, a balance between goodness of fit and simplicity of model is sought. For polynomials, we try to pick a model with the lowest possible degree that has a reasonably good fit.

QUICK REVIEW 2.3 *(For help, go to Sections A.2. and P.5.)*

In Exercises 1–6, factor the polynomial into linear factors.

1. $x^2 - x - 12$
2. $x^2 - 11x + 28$
3. $3x^2 - 11x + 6$
4. $6x^2 - 5x + 1$
5. $3x^3 - 5x^2 + 2x$
6. $6x^3 - 22x^2 + 12x$

In Exercises 7–10, solve the equation mentally.

7. $x(x - 1) = 0$
8. $x(x + 2)(x - 5) = 0$
9. $(x + 6)^3(x + 3)(x - 1.5) = 0$
10. $(x + 6)^2(x + 4)^4(x - 5)^3 = 0$

SECTION 2.3 EXERCISES

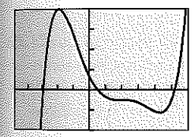
In Exercises 1–6, describe how to transform the graph of an appropriate monomial function $f(x) = x^n$ into the graph of the given polynomial function. Sketch the transformed graph by hand and support your answer with a grapher. Compute the location of the y-intercept as a check on the transformed graph.

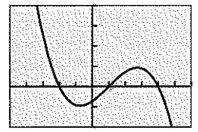
1. $g(x) = 2(x - 3)^3$
2. $g(x) = -(x + 5)^3$
3. $g(x) = -\frac{1}{2}(x + 1)^3 + 2$
4. $g(x) = \frac{2}{3}(x - 3)^3 + 1$
5. $g(x) = -2(x + 2)^4 - 3$
6. $g(x) = 3(x - 1)^4 - 2$

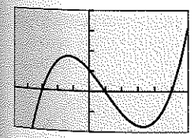
In Exercises 7 and 8, graph the polynomial function, locate its extrema and zeros, and explain how it is related to the monomials from which it is built.

7. $f(x) = -x^4 + 2x$
8. $g(x) = 2x^4 - 5x^2$

In Exercises 9–12, match the polynomial function with its graph. Explain your choice. Do not use a graphing calculator.


[−5, 6] by [−200, 400]
(a)


[−5, 6] by [−200, 400]
(b)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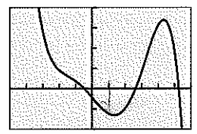

[−5, 6] by [−200, 400]
(d)

9. $f(x) = 7x^3 - 21x^2 - 91x + 104$
10. $f(x) = -9x^3 + 27x^2 + 54x - 73$
11. $f(x) = x^5 - 8x^4 + 9x^3 + 58x^2 - 164x + 69$
12. $f(x) = -x^5 + 3x^4 + 16x^3 - 2x^2 - 95x - 44$

In Exercises 13–16, graph the function pairs in the same series of viewing windows. Zoom out until the two graphs look nearly identical and state your final viewing window.

13. $f(x) = x^3 - 4x^2 - 5x - 3$ and $g(x) = x^3$
14. $f(x) = x^3 + 2x^2 - x + 5$ and $g(x) = x^3$
15. $f(x) = 2x^3 + 3x^2 - 6x - 15$ and $g(x) = 2x^3$
16. $f(x) = 3x^3 - 12x + 17$ and $g(x) = 3x^3$

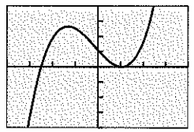
In Exercises 17–24, graph the function in a viewing window that shows all of its extrema and x-intercepts. Describe the end behavior using limits.

17. $f(x) = (x - 1)(x + 2)(x + 3)$
18. $f(x) = (2x - 3)(4 - x)(x + 1)$
19. $f(x) = -x^3 + 4x^2 + 31x - 70$
20. $f(x) = x^3 - 2x^2 - 41x + 42$
21. $f(x) = (x - 2)^2(x + 1)(x - 3)$
22. $f(x) = (2x + 1)(x - 4)^3$
23. $f(x) = 2x^4 - 5x^3 - 17x^2 + 14x + 41$
24. $f(x) = -3x^4 - 5x^3 + 15x^2 - 5x + 19$

In Exercises 25–28, describe the end behavior of the polynomial function using $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

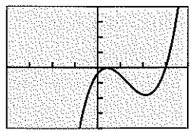
25. $f(x) = 3x^4 - 5x^2 + 3$
26. $f(x) = -x^3 + 7x^2 - 4x + 3$
27. $f(x) = 7x^2 - x^3 + 3x - 4$
28. $f(x) = x^3 - x^4 + 3x^2 - 2x + 7$

In Exercises 29–32, match the polynomial function with its graph. Approximate all of the real zeros of the function.



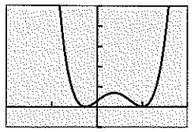
[–4, 4] by [–200, 200]

(a)



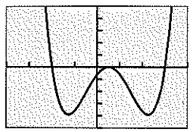
[–4, 4] by [–200, 200]

(b)



[–2, 2] by [–10, 50]

(c)



[–4, 4] by [–50, 50]

(d)

29. $f(x) = 20x^3 + 8x^2 - 83x + 55$

30. $f(x) = 35x^3 - 134x^2 + 93x - 18$

31. $f(x) = 44x^4 - 65x^3 + x^2 + 17x + 3$

32. $f(x) = 4x^4 - 8x^3 - 19x^2 + 23x - 6$

In Exercises 33–38, find the zeros of the function algebraically.

33. $f(x) = x^2 + 2x - 8$ **34.** $f(x) = 3x^2 + 4x - 4$

35. $f(x) = 9x^2 - 3x - 2$ **36.** $f(x) = x^3 - 25x$

37. $f(x) = 3x^3 - x^2 - 2x$ **38.** $f(x) = 5x^3 - 5x^2 - 10x$

In Exercises 39–42, state the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of the polynomial function by hand.

39. $f(x) = x(x - 3)^2$ **40.** $f(x) = -x^3(x - 2)$

41. $f(x) = (x - 1)^3(x + 2)^2$ **42.** $f(x) = 7(x - 3)^2(x + 5)^4$

In Exercises 43–48, graph the function in a viewing window that shows all of its x-intercepts and approximate all of its zeros.

43. $f(x) = 2x^3 + 3x^2 - 7x - 6$

44. $f(x) = -x^3 + 3x^2 + 7x - 2$

45. $f(x) = x^3 + 2x^2 - 4x - 7$

46. $f(x) = -x^4 - 3x^3 + 7x^2 + 2x + 8$

47. $f(x) = x^4 + 3x^3 - 9x^2 + 2x + 3$

48. $f(x) = 2x^5 - 11x^4 + 4x^3 + 47x^2 - 42x - 8$

In Exercises 49–52, find the zeros of the function algebraically or graphically.

49. $f(x) = x^3 - 36x$

50. $f(x) = x^3 + 2x^2 - 109x - 110$

51. $f(x) = x^3 - 7x^2 - 49x + 55$

52. $f(x) = x^3 - 4x^2 - 44x + 96$

In Exercises 53–56, using only algebra, find a cubic function with the given zeros. Support by graphing your answer.

53. $3, -4, 6$ **54.** $-2, 3, -5$

55. $\sqrt{3}, -\sqrt{3}, 4$ **56.** $1, 1 + \sqrt{2}, 1 - \sqrt{2}$

57. Use cubic regression to fit a curve through the four points given in the table.

x	-3	-1	1	3
y	22	25	12	-5

58. Use cubic regression to fit a curve through the four points given in the table.

x	-2	1	4	7
y	2	5	9	26

59. Use quartic regression to fit a curve through the five points given in the table.

x	3	4	5	6	8
y	-7	-4	-11	8	3

60. Use quartic regression to fit a curve through the five points given in the table.

x	0	4	5	7	13
y	-21	-19	-12	8	3

In Exercises 61–62, explain why the function has at least one real zero.

61. Writing to Learn $f(x) = x^7 + x + 100$

62. Writing to Learn $f(x) = x^9 - x + 50$

63. Stopping Distance A state highway patrol safety division collected the data on stopping distances in Table 2.14 on the next page.

(a) Draw a scatter plot of the data.

(b) Find the quadratic regression model.

(c) Superimpose the regression curve on the scatter plot.

(d) Use the regression model to predict the stopping distance for a vehicle traveling at 25 mph.

(e) Use the regression model to predict the speed of a car if the stopping distance is 300 ft.

Table 2.14 Highway Safety Division

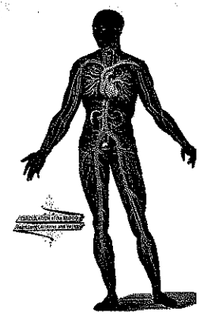
Speed (mph)	Stopping Distance (ft)
10	15.1
20	39.9
30	75.2
40	120.5
50	175.9

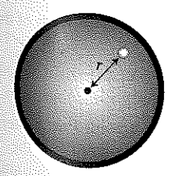
64. Analyzing Profit Economists for Smith Brothers, Inc., find the company profit P by using the formula $P = R - C$, where R is the total revenue generated by the business and C is the total cost of operating the business.

(a) Using data from past years, the economists determined that $R(x) = 0.0125x^2 + 412x$ models total revenue, and $C(x) = 12{,}225 + 0.00135x^3$ models the total cost of doing business, where x is the number of customers patronizing the business. How many customers must Smith Bros. have to be profitable each year?

(b) How many customers must there be for Smith Bros. to realize an annual profit of $60,000?

65. Circulation of Blood Research conducted at a national health research project shows that the speed at which a blood cell travels in an artery depends on its distance from the center of the artery. The function $v = 1.19 - 1.87r^2$ models the velocity (in centimeters per second) of a cell that is r centimeters from the center of an artery.





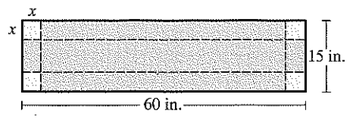
(a) Find a graph of v that reflects values of v appropriate for this problem. Record the viewing-window dimensions.

(b) If a blood cell is traveling at 0.975 cm/sec, estimate the distance the blood cell is from the center of the artery.

66. Volume of a Box Dixie Packaging Co. has contracted to manufacture a box with no top that is to be made by removing squares of width x from the corners of a 15-in. by 60-in. piece of cardboard.

(a) Show that the volume of the box is modeled by $V(x) = x(60 - 2x)(15 - 2x)$.

(b) Determine x so that the volume of the box is at least 450 in.^{3}



67. Volume of a Box Squares of width x are removed from a 10-cm by 25-cm piece of cardboard, and the resulting edges are folded up to form a box with no top. Determine all values of x so that the volume of the resulting box is at most 175 cm^3.

68. Volume of a Box The function $V = 2666x - 210x^2 + 4x^3$ represents the volume of a box that has been made by removing squares of width x from each corner of a rectangular sheet of material and then folding up the sides. What values are possible for x?

Standardized Test Questions

69. True or False The graph of $f(x) = x^3 - x^2 - 2$ crosses the x-axis between $x = 1$ and $x = 2$. Justify your answer.

70. True or False If the graph of $g(x) = (x + a)^2$ is obtained by translating the graph of $f(x) = x^2$ to the right, then a must be positive. Justify your answer.

In Exercises 71 and 72, solve the problem without using a calculator.

71. Multiple Choice What is the y-intercept of the graph of $f(x) = 2(x - 1)^3 + 5$?

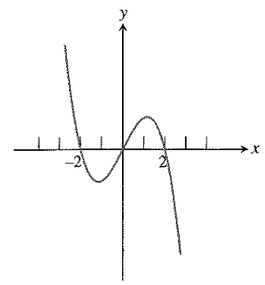
(A) 7 **(B)** 5 **(C)** 3 **(D)** 2 **(E)** 1

72. Multiple Choice What is the multiplicity of the zero $x = 2$ in $f(x) = (x - 2)^2(x + 2)^3(x + 3)^7$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

In Exercises 73 and 74, which of the specified functions might have the given graph?

73. Multiple Choice



(A) $f(x) = -x(x + 2)(2 - x)$

(B) $f(x) = -x(x + 2)(x - 2)$

(C) $f(x) = -x^2(x + 2)(x - 2)$

(D) $f(x) = -x(x + 2)^2(x - 2)$

(E) $f(x) = -x(x + 2)(x - 2)^2$